Clock Metrics

WSTS 2021 Tutorial Session

March 24, 2021

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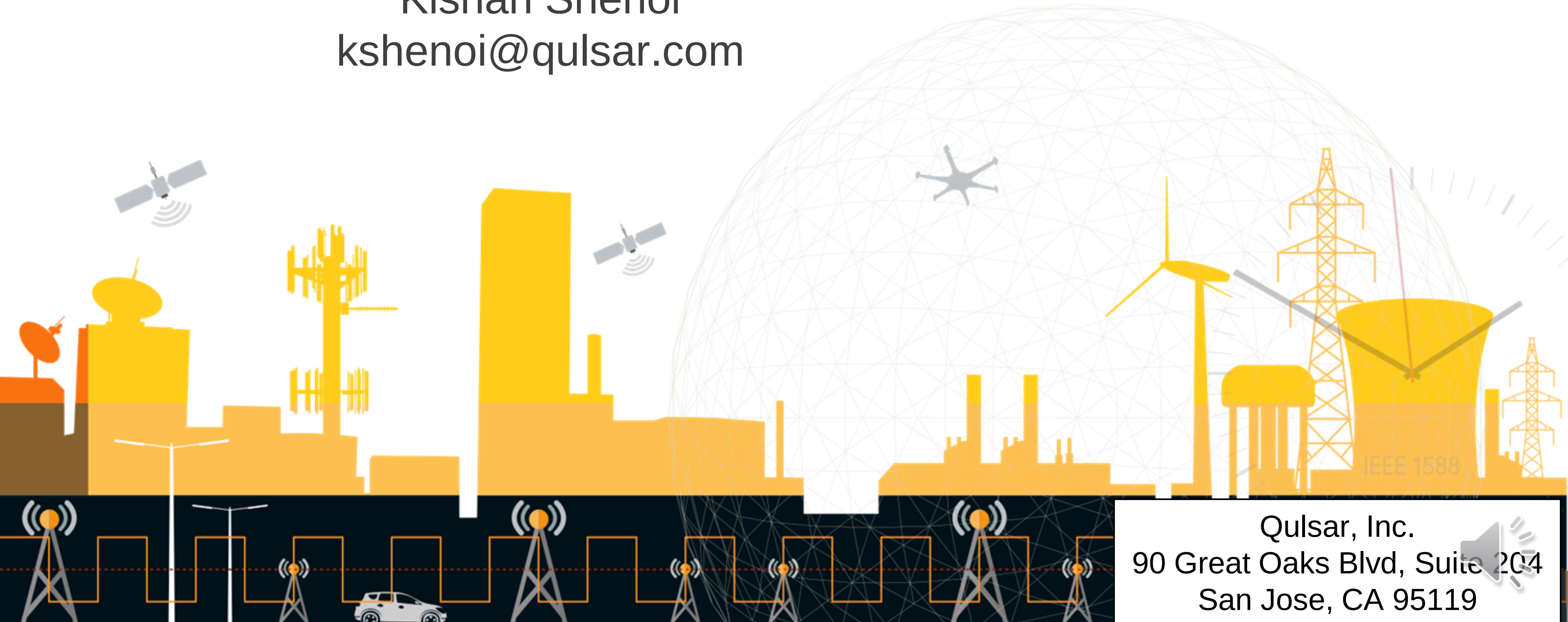

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Synchronization Metrics (Performance)

- Mathematical Model
- Fundamental Clock Concepts and Metrics
  - Time Error (TE) and Time Interval Error (TIE)
  - MTIE
  - TDEV
- Relationship between TDEV, Spectrum, and MTIE
  - Use of TDEV to identify noise type
  - Using TDEV for guidance on loop bandwidths

Common Mathematical Models

$$clock(t) = A \cdot \cos\left(\Phi(t)\right) = A \cdot \cos\left(\omega \cdot t + \phi_0 + \varepsilon(t)\right)$$

signal

Mathematical time

Phase function

(radian) frequency

"Clock Noise"

- A: Amplitude of signal. Does not figure in timing metrics.
- ϕ_0: Initial phase. Depends on choice of time origin. Usually assumed to be 0.
- $\varepsilon(t)$: Can be further decomposed into different categories such as frequency error, frequency drift, and random noise components
- ideal periodic signal: $\Phi(t)$ is a linear function of t ($\varepsilon(t) \equiv 0$)

Continuous time view

$$x(t) = a_0 + y \cdot t + \left(\frac{1}{2}\right) \cdot D \cdot t^2 + \phi(t)$$

Discrete time view

$$x(nT_s) = a_0 + y \cdot nT_s + \left(\frac{1}{2}\right) \cdot D \cdot (nT_s)^2 + \phi(nT_s)$$

Time Error Models

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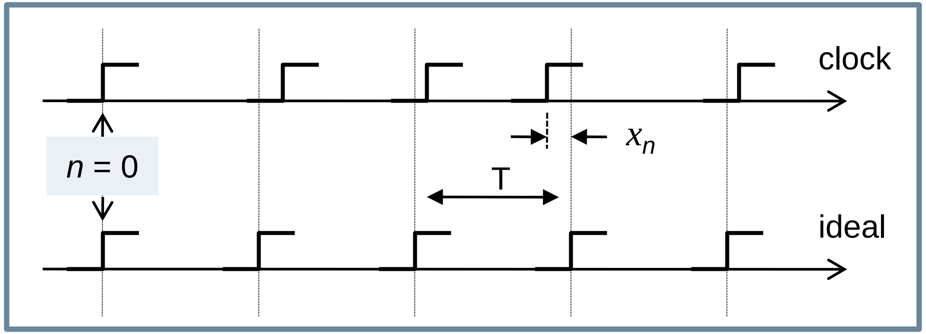
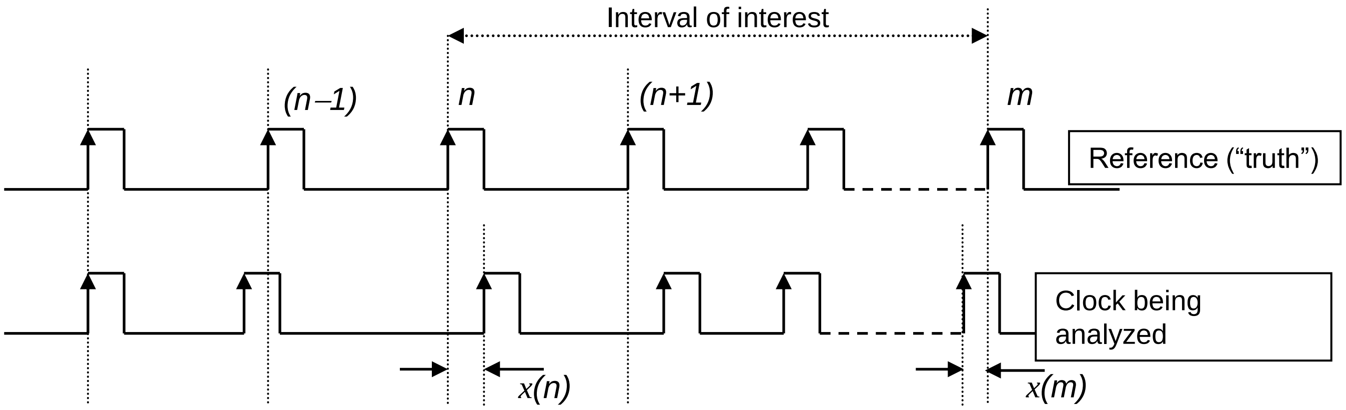
Clock Metrics – Basics: Time Error



- Clock signals are (<u>almost</u>) periodic (<u>nominal</u> period ~ T)
- Time Error (Phase Error):
  - Edges do not line up – *phase* error (expressed in time units)
- Time Error Sequence : $\{x_n\}$ or $\{x(n)\}$
  - *All clock metrics derived from time error sequence*
  - Note: the time error varies "slowly" so we can divide down to a convenient rate (However: careful when dividing down – aliasing)
  - Common assumption: $x_0 = 0$.

Time Interval Error



- Consider an interval of interest (e.g. 100m dash)
- Duration measured by ideal clock ("truth") : $(m - n) \cdot T_S$
- *Error* in measurement of same interval by clock being analyzed:

$$TIE(m,n) = x(m) - x(n)$$

Clock Metrics – MTIE and TDEV

MTIE

A measure of peak-to-peak excursion expected within a given interval, τ (τ is a parameter). The observation interval is scanned with a moving window of duration τ and MTIE(τ) is the maximum excursion.

Given a set of N observations $\{x(k);\ k=0,1,2,\ldots,(N\text{-}1)\}$, with underlying sampling interval τ_0, let $\tau = n \cdot \tau_0$ ("window" = n samples; $n = 1,2,\ldots,N$).

Peak-to-peak excursion over n samples starting with sample index i is the worst-case TIE in this interval of n samples:

$$peak-to-peak(i) = \{ \max_{k=i}^{k=i+n-1} x(k) - \min_{k=i}^{k=i+n-1} x(k) \}$$

MTIE(n), or MTIE(τ), is the largest value of this peak-to-peak excursion:

$$MTIE(n) = \max_{i=0}^{N\text{-}n} \{ \max_{k=i}^{k=i+n-1} x(k) - \min_{k=i}^{k=i+n-1} x(k) \}$$

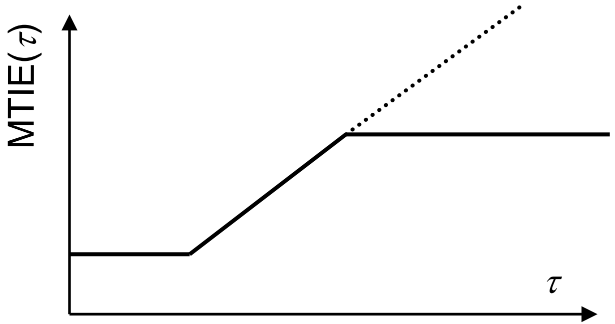
Clock Metrics – MTIE and TDEV

MTIE

MTIE is a useful indicator of the size of buffers and for predicting buffer overflows and underflows.

Buffer size > MTIE(τ) implies that overflow/underflow unlikely in any interval < τ

Buffer size = MTIE(τ) implies that overflow/underflow occurs approx. every τ seconds



Observations regarding MTIE:

- monotonically increasing with τ
- linear increase indicates freq. offset
- for small τ, MTIE(τ) $\leftrightarrow$ jitter
- for medium τ, MTIE(τ) $\leftrightarrow$ wander
- for large τ, indicates whether “locked” (zero-slope)

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Clock Metrics – MTIE and TDEV

TDEV

A measure of stability expected over a given observation interval, τ (τ is a parameter).

Given a set of N observations $\{x(k);\ k=0,1,2,\ldots,(N-1)\}$ with underlying sampling interval τ_0, let $\tau = n\cdot\tau_0$ ("window" = n samples; $n = 1,2,\ldots,N$).

Note: $x(k) \Leftrightarrow x_k$

$$\sigma_x(\tau) = TDEV(\tau) = \sqrt{\frac{1}{6n^2(N-3n+1)} \sum_{j=0}^{N-3n} \left[\sum_{i=j}^{n+j-1} \left(x_{i+2n} - 2x_{i+n} + x_i \right) \right]^2}$$

$$\text{for } n = 1,2,\ldots,\left\lfloor \frac{N}{3} \right\rfloor$$

Conventional Definition

- Second-order difference: $\left(x_{i+2n} - 2x_{i+n} + x_i \right)$
- N-point averaging: $\sum_{i=j}^{n+j-1}$
- Sum of squares: $\sum_{j=0}^{N-3n}$

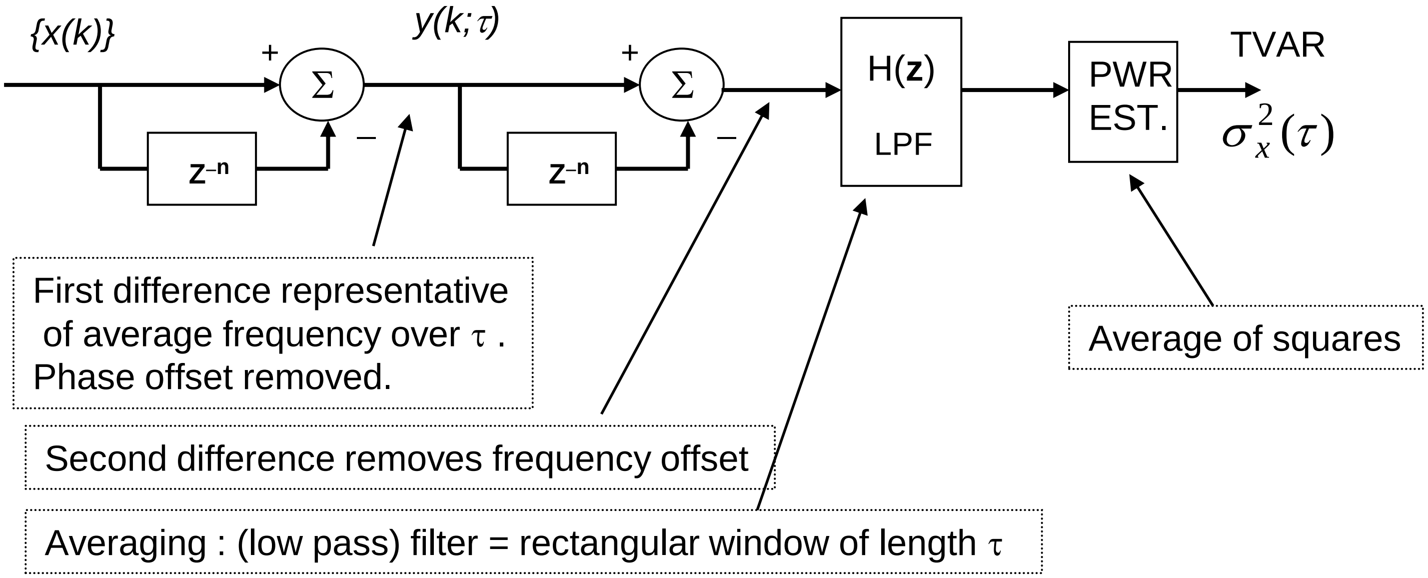
- TVAR = square of TDEV
- Modified Allan Deviation (MDEV)

$$\sigma_y(\tau) = \frac{\sqrt{3}}{\tau}\sigma_x(\tau)$$

- TDEV suppresses initial phase and frequency offset and quantifies the strength of the frequency drift and noise components {i.e. $\varepsilon(t)$}
- TDEV provides guidance on the noise process type

Clock Metrics – MTIE and TDEV

Signal Processing Interpretation of TDEV and TVAR



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Noise Types, TDEV, Spectrum & MTIE

TDEV (and MDEV) for different noise types

Noise Process	Dependence of TDEV(τ) on τ	Dependence of MDEV(τ) on τ
White PM	$\tau^{-(1/2)}$	$\tau^{-(3/2)}$
Flicker PM	τ^{0}	τ^{-1}
Random Walk PM = White FM	$\tau^{+(1/2)}$	$\tau^{-(1/2)}$
Flicker FM	τ^{+1}	τ^{0}
Random Walk FM	$\tau^{+(3/2)}$	$\tau^{+(1/2)}$

When linear frequency drift dominates, TDEV(τ) behaves as τ^2

Spectrum (*S(f)*)for different noise types

Noise Process	Spectrum Type (power)
White PM	f^{0}
Flicker PM	f^{-1}
Random Walk PM = White FM	f^{-2}
Flicker FM	f^{-3}
Random Walk FM	f^{-4}

Approximate relationship between TDEV and power spectrum:
(For guidance purposes only)

$$S_x(f) \approx \left(\frac{0.75}{f}\right) \cdot \left(\sigma_x\left(\frac{0.3}{f}\right)\right)^2$$

$$\sigma_x(\tau) \approx \sqrt{\left(\frac{1}{2.5 \cdot \tau}\right) \cdot S_x\left(\frac{0.3}{\tau}\right)}$$

Approximate relationship between TDEV and MTIE:
$K_1 \sim 0.75$; $K_2 \sim 0.3$ (excludes effect of transients)

$$M_x(\tau) \le 7 \cdot \sqrt{4 \cdot K_1 \cdot \int_0^{f_0} \frac{1}{f} \cdot TVAR\left(\frac{K_2 \cdot f_0}{f}\right) \cdot \sin^2(n\pi f \tau_0) \cdot df}$$

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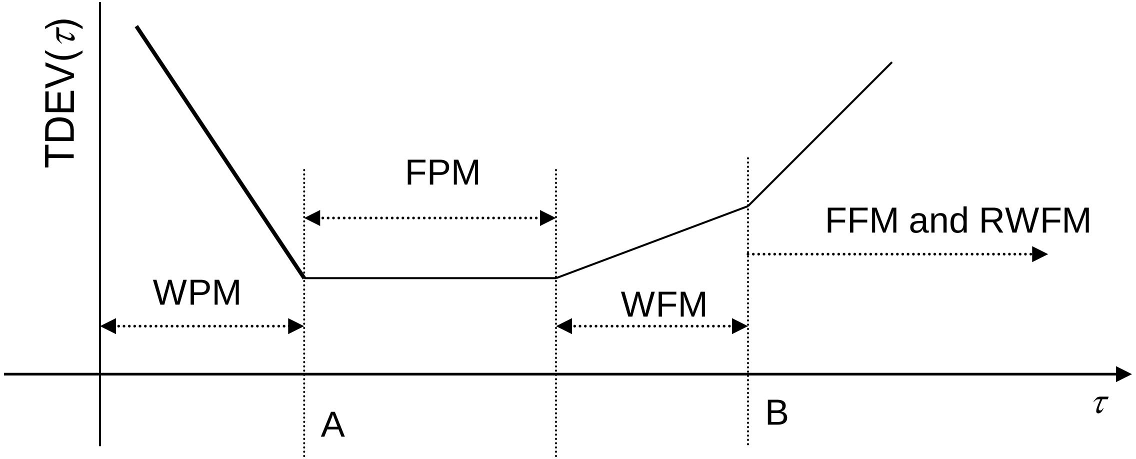
Implication of TDEV(τ) versus τ



"Phase coherence" for up to A sec.
$\Rightarrow$ Keep PLL time constants less than A sec.

"Frequency coherence" for up to B sec.
$\Rightarrow$ Keep FLL time constants less than B sec.

Phase Flicker Floor

Frequency Flicker Floor

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Thank You

Questions, comments, suggestions?

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