Assisted Partial Timing Support: Predicting Time Dispersion

Aka: Optimal Estimation, Prediction, and Ensembling Of Timing Signals

Marc Weiss, Ph.D.

NIST Consultant

Optimal: Outline

- Timing Signal Model
  - Model
  - Power Law Spectra
  - Linear Frequency Drift
- Optimal Estimation and Prediction
- Optimal Ensembling of Signals

Clock/System Model

$$x(t) = x_0 + y_0 t + D\frac{t^2}{2} + \varepsilon(t)$$

where: $x(t)$ = time deviation after time t

systematic or deterministic part

x_0= initial time offset

y_0= initial frequency offset

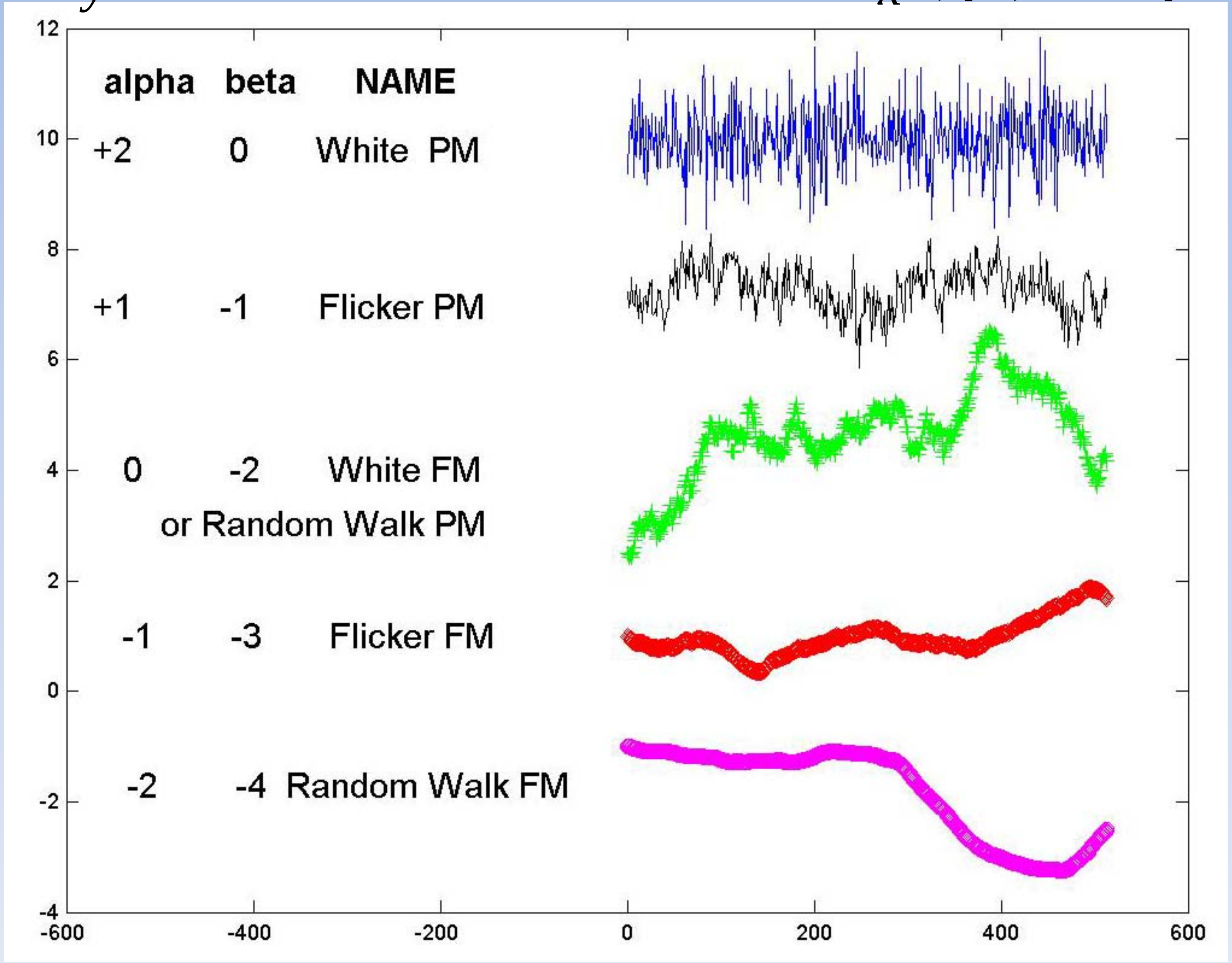
D= linear frequency drift

random or stochastic part

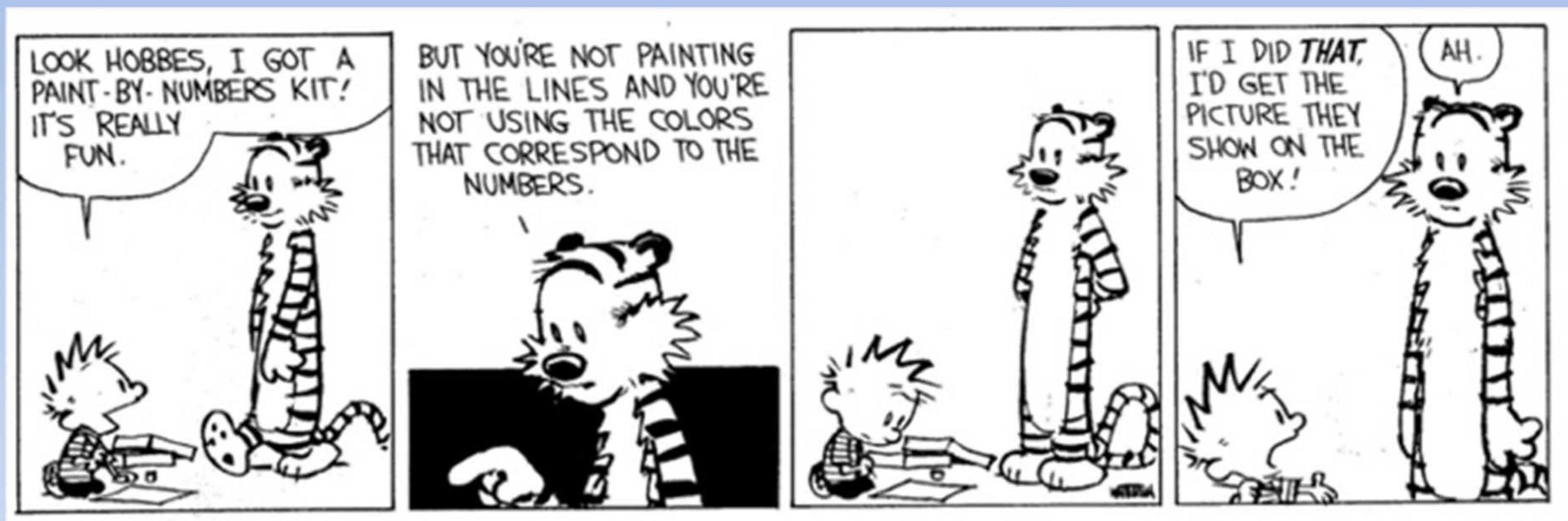
$\varepsilon(t)$= white, flicker, or random walk phase or frequency modulation, or combinations

POWER-LAW SPECTRA:

$$S_y(f) \propto f^{\alpha} \qquad\qquad S_x(f) \propto f^{\beta}$$

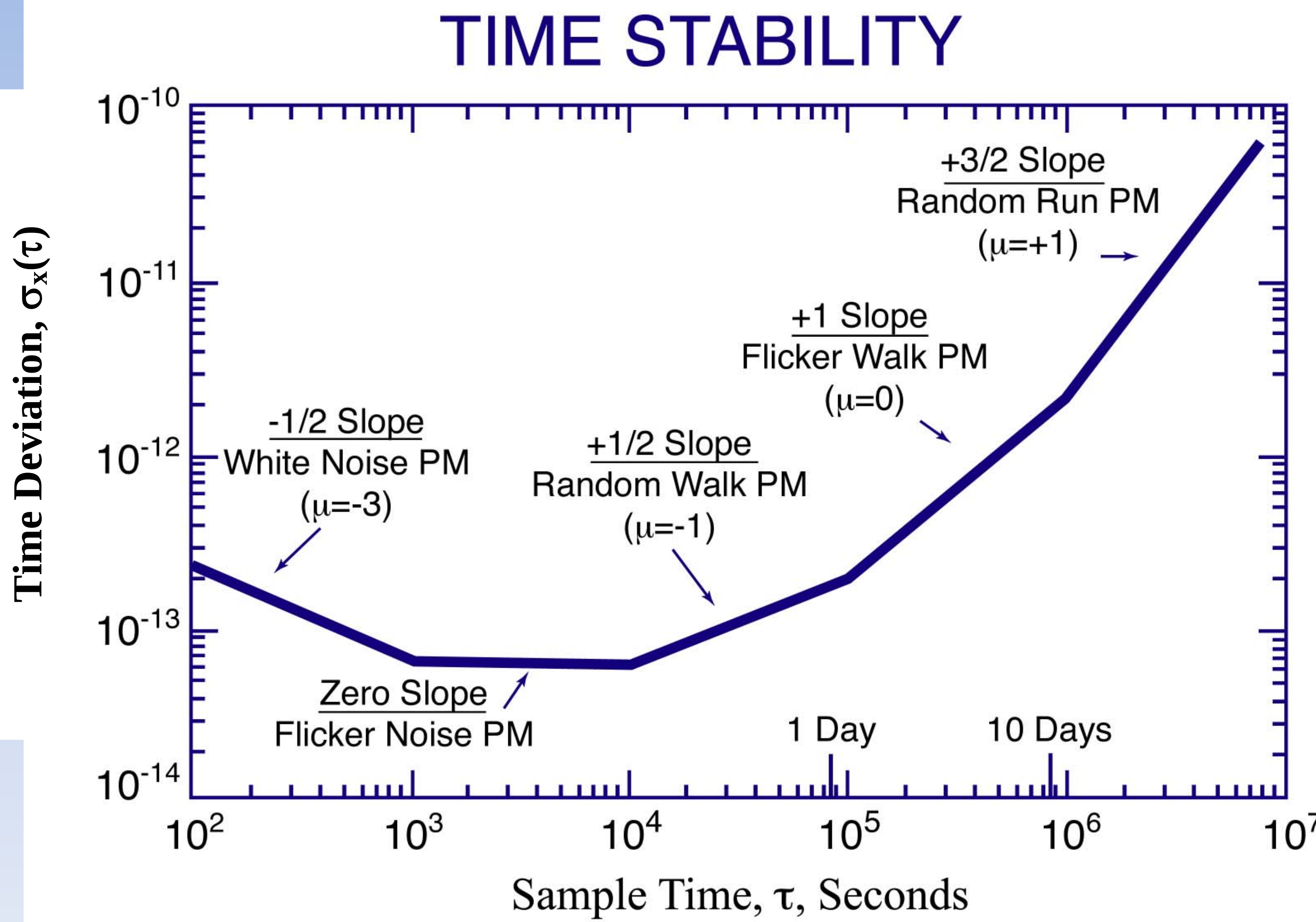


There are many types of “Random”



The Standard Deviation may not mean anything.

TDEV Reveals the Noise Type



Linear Frequency Drift

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

Estimating Linear Frequency Drift: Motivation

- Linear Frequency Drift Biases AVAR, MVAR and TVAR
  - Affects characterization of stochastic noise type
  - Affects confidence intervals
  - Alternative: use Hadamard variance
- Time Prediction Error Grows as τ^2
- Optimal Estimate of Drift a Function of Noise Type
  - Depends on judgment
  - Requires extrapolation of variance to data length

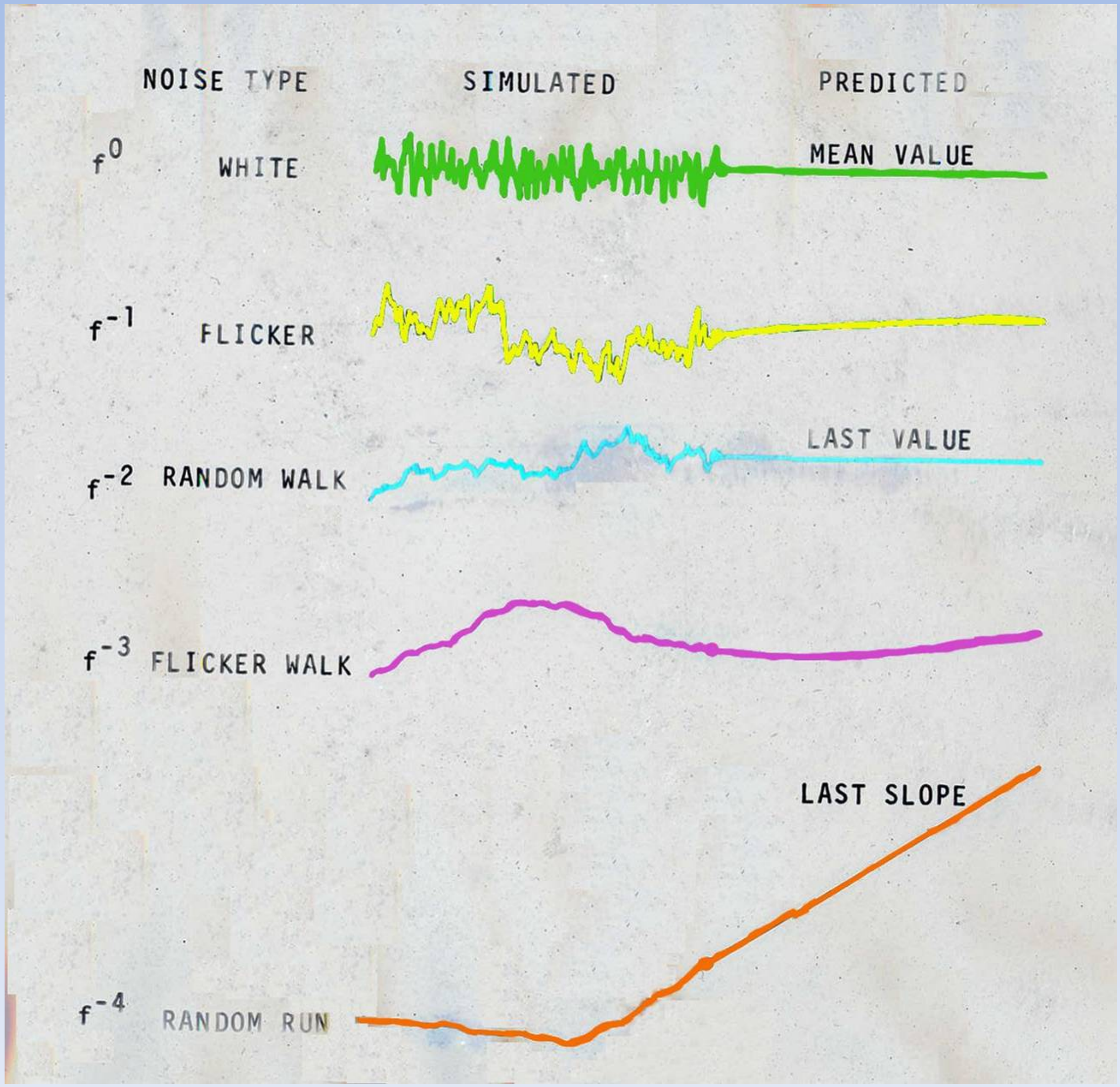
Optimal: Outline

- Timing Signal Model
  - Model
  - Power Law Spectra
  - Linear Frequency Drift
- Optimal Estimation and Prediction
- Optimal Ensembling of Signals

Optimal Prediction Theory

- Optimal estimate/ prediction of White PM is the Mean
- General Method
  - Filter data to make it White PM
  - Take Mean
  - Reverse filter the mean

Optimal Predictors by Noise Type



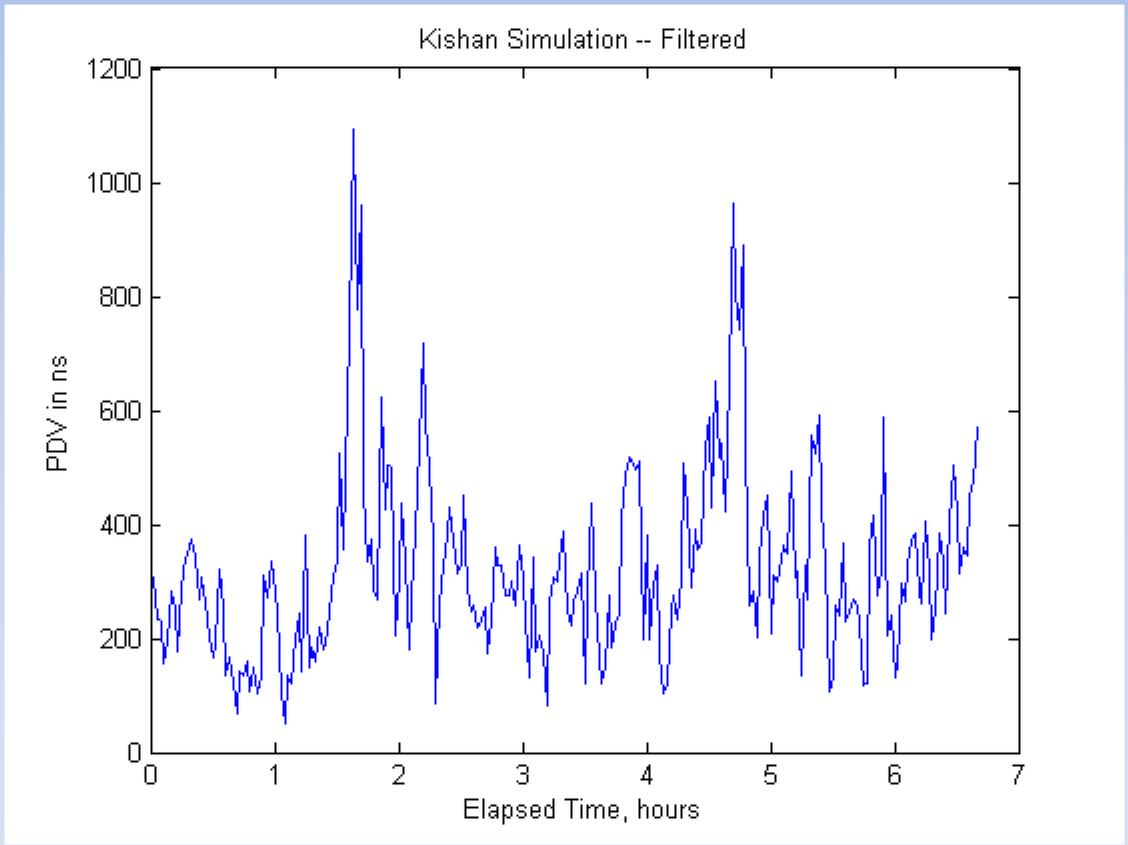
Expected Time Dispersion: Optimum Prediction is Based on Noise Types

These expressions are in terms of the Allan Deviation

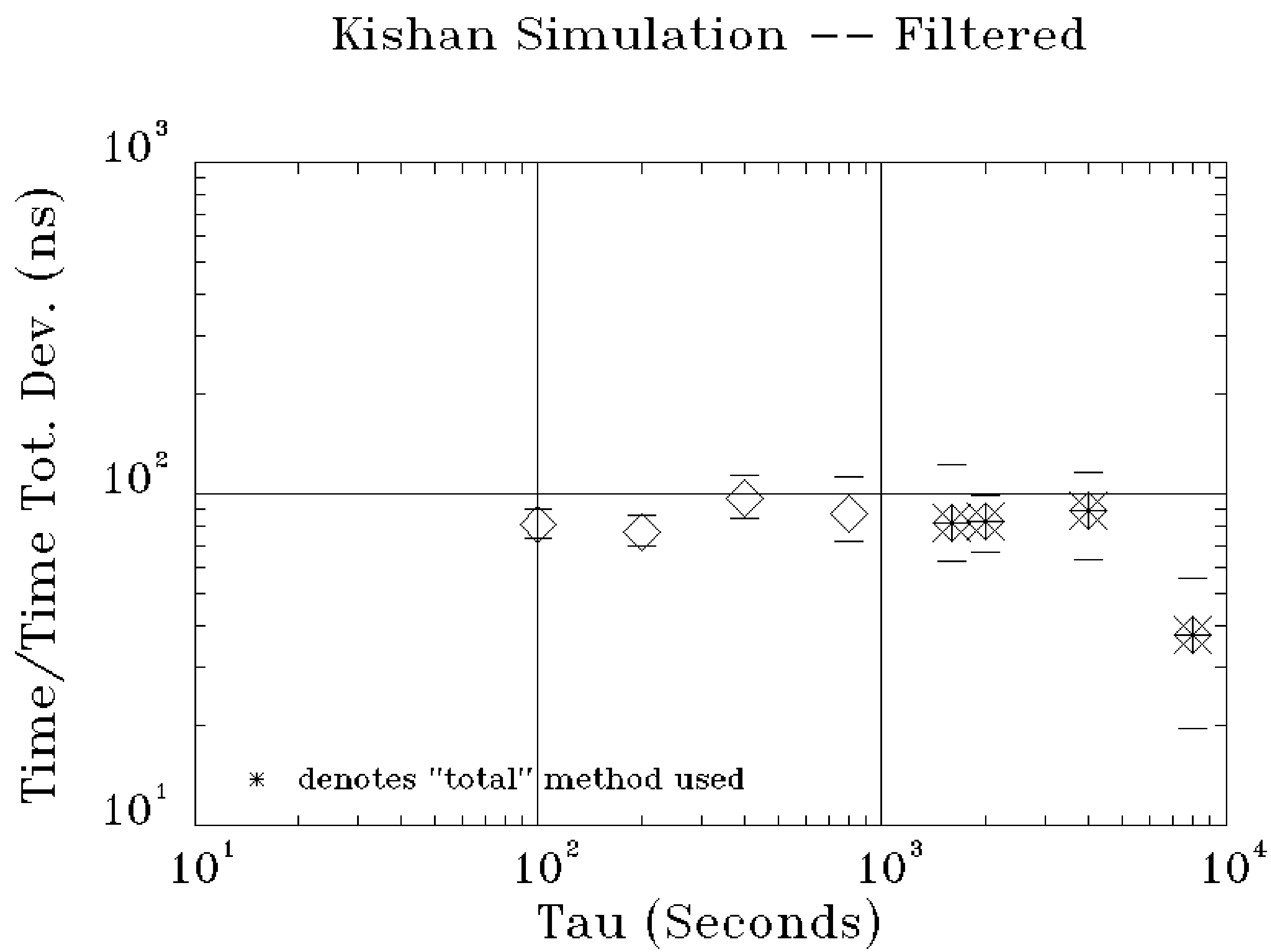
α	Typical Noise Types Name	Optimum Prediction $x(\tau_p)$ rms[a]	Time Error: Asymptotic Form
2	white-noise PM	$\tau_p \cdot \sigma_y(\tau_p)/\sqrt{3}$	constant
1	flicker-noise PM	$\sim \tau_p \cdot \sigma_y(\tau_p)\sqrt{\ln \tau_p / 2 \ln \tau_0}$	$\sqrt{\ln \tau_p}$
0	white-noise FM	$\tau_p \cdot \sigma_y(\tau_p)$	$\tau_p^{1/2}$
−1	flicker-noise FM	$\tau_p \cdot \sigma_y(\tau_p)/\sqrt{\ln 2}$	τ_p
−2	random-walk FM	$\tau_p \cdot \sigma_y(\tau_p)$	$\tau_p^{3/2}$

[a] τ_p is the prediction interval.

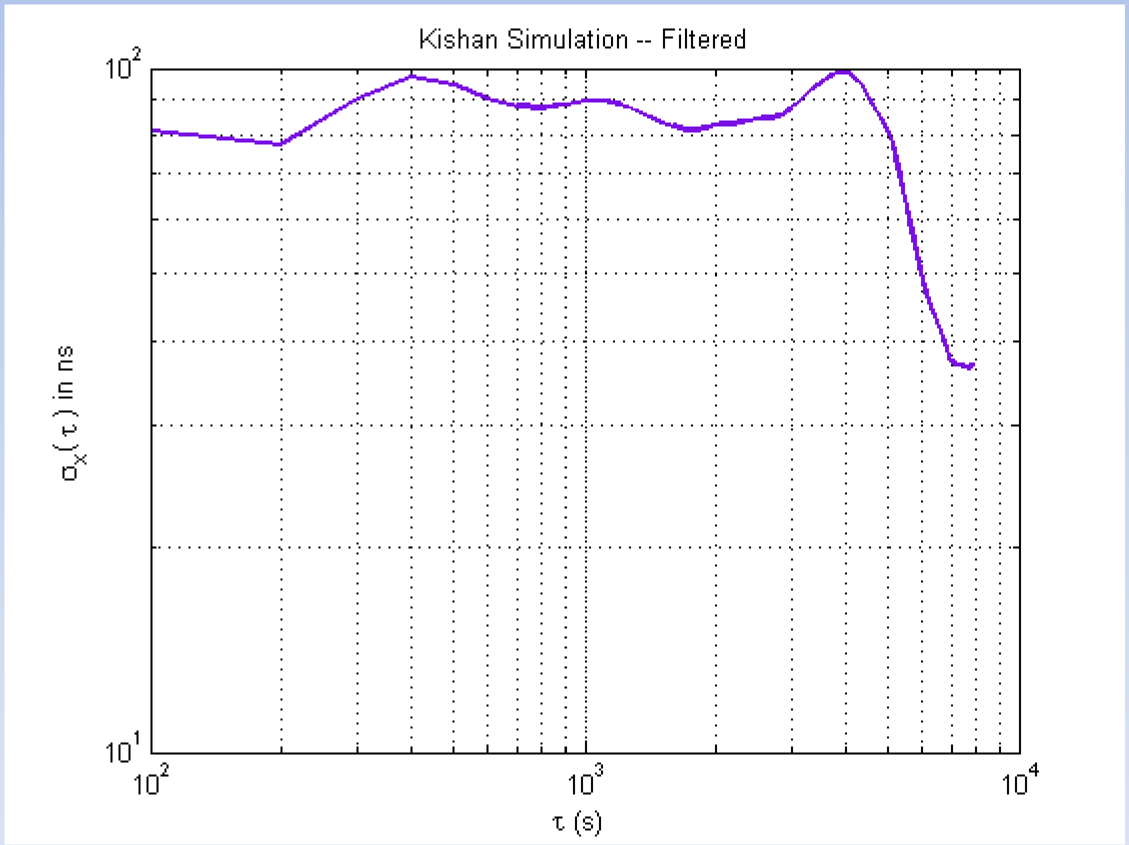
An Example of Optimal Prediction on Kishan's Simulated, Filtered Data



TDEV at tau = 2^n
Showing Uncertainty



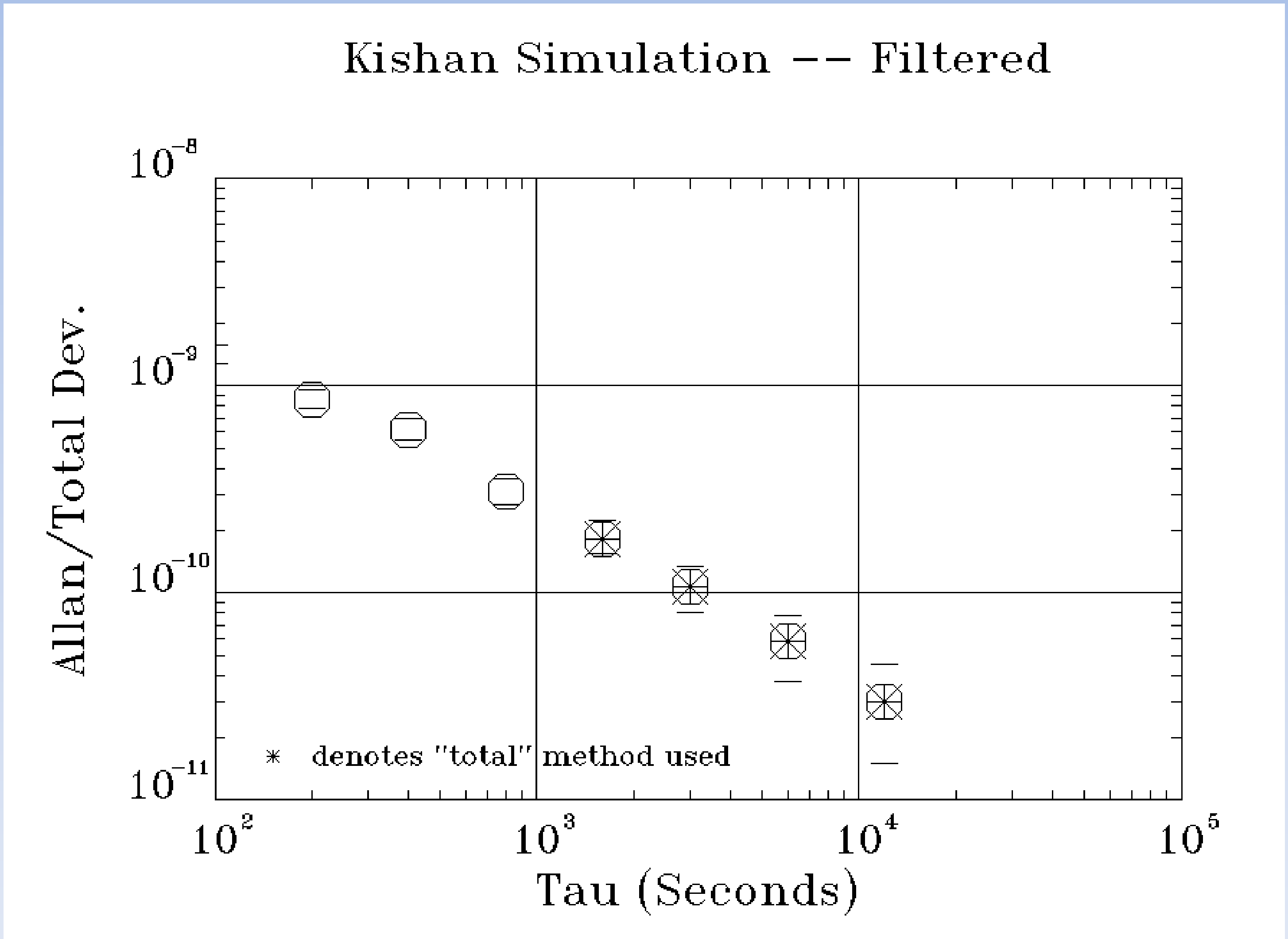
TDEV at All tau Values: No evident Periodic Behavior

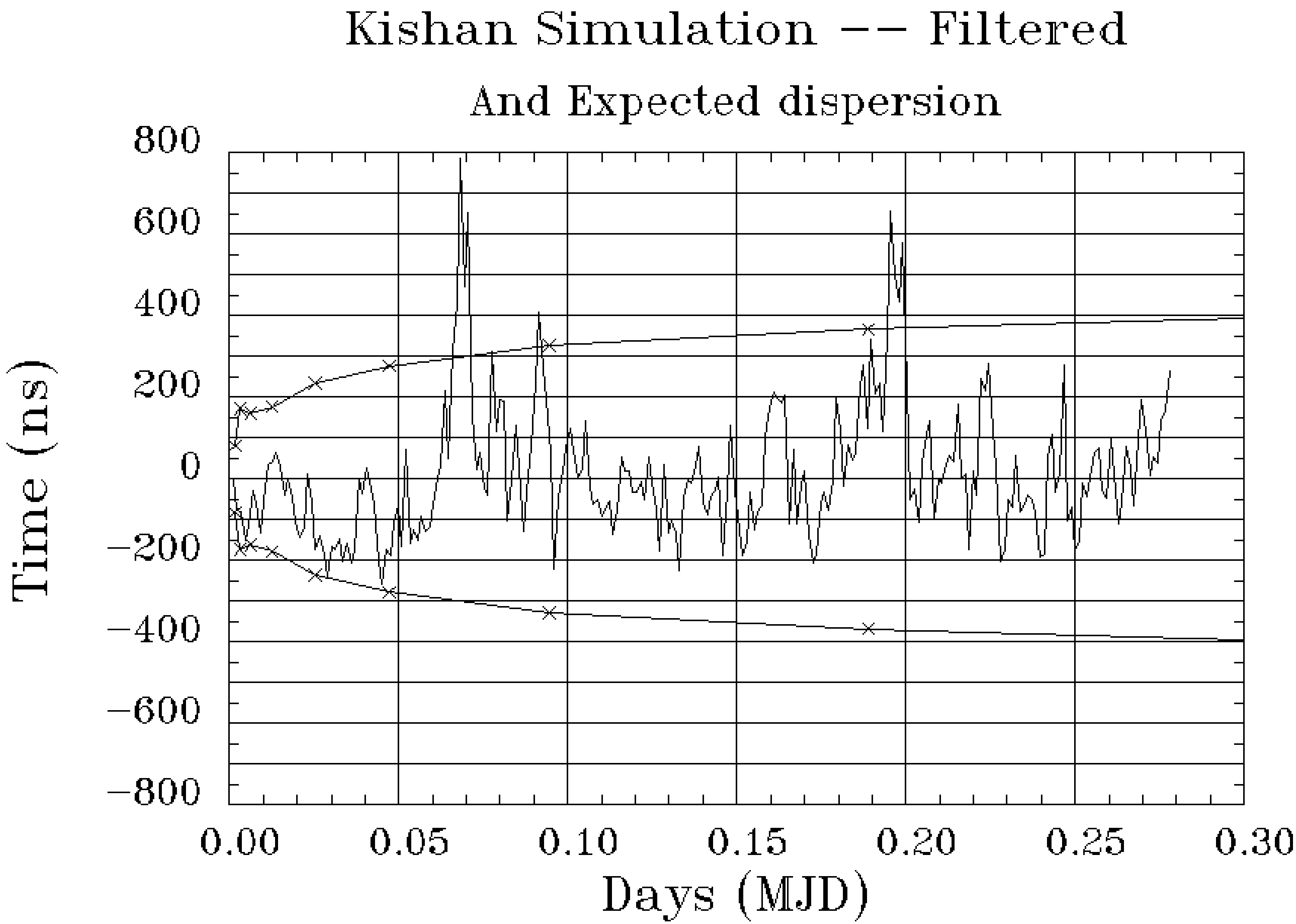


Kishan Simulation – Filtered
Noise Types Automatically Detected

m	Tau(s)	Deviation	Lower	Upper	Noise Type
1	100	81.0	73.4	90.6	White PM
2	200	77.2	69.7	86.7	White FM
4	400	96.9	84.2	11.5	Flicker PM
8	800	87.4	72.0	11.2	Flicker PM
16	1600	82.1	62.6	12.2	Flicker PM
20	2000	82.9	66.6	99.2	Flicker PM
40	4000	89.5	63.2	115.9	Flicker PM
80	8000	37.7	19.7	55.7	Flicker PM

Allan Deviation for Optimal Prediction Equations





Kishan Simulation -- Filtered
And Expected dispersion
800
600
400
200
0
−200
−400
−600
−800
Time (ns)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Days (MJD)

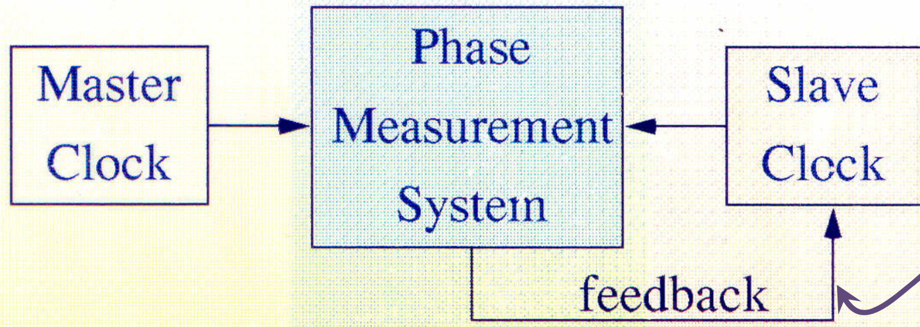
Optimal: Outline

- Timing Signal Model
  - Model
  - Power Law Spectra
  - Linear Frequency Drift
- Optimal Estimation and Prediction
- Optimal Ensembling of Signals

Optimal Ensembling

- With two clocks: Phase lock loop
- With multiple clocks: Time Scale Algorithm

Generic Phase-lock Loop



Slave averages master signal over the lock time

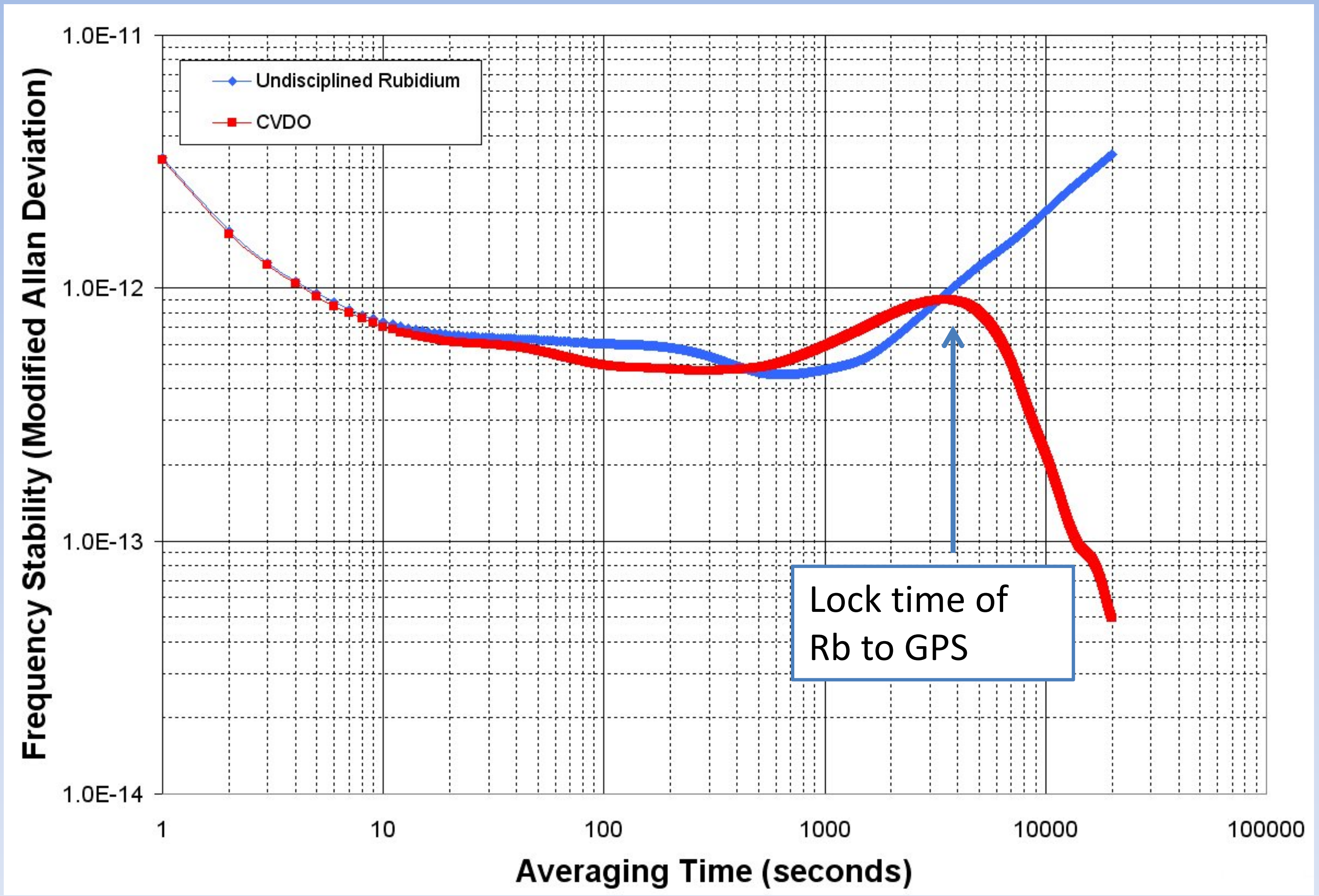
Fundamental Properties

$$y = \frac{\nu - \nu_0}{\nu_0} \qquad x = \int y' \, dt' = \frac{\phi}{2\pi\nu_0} \qquad y$$

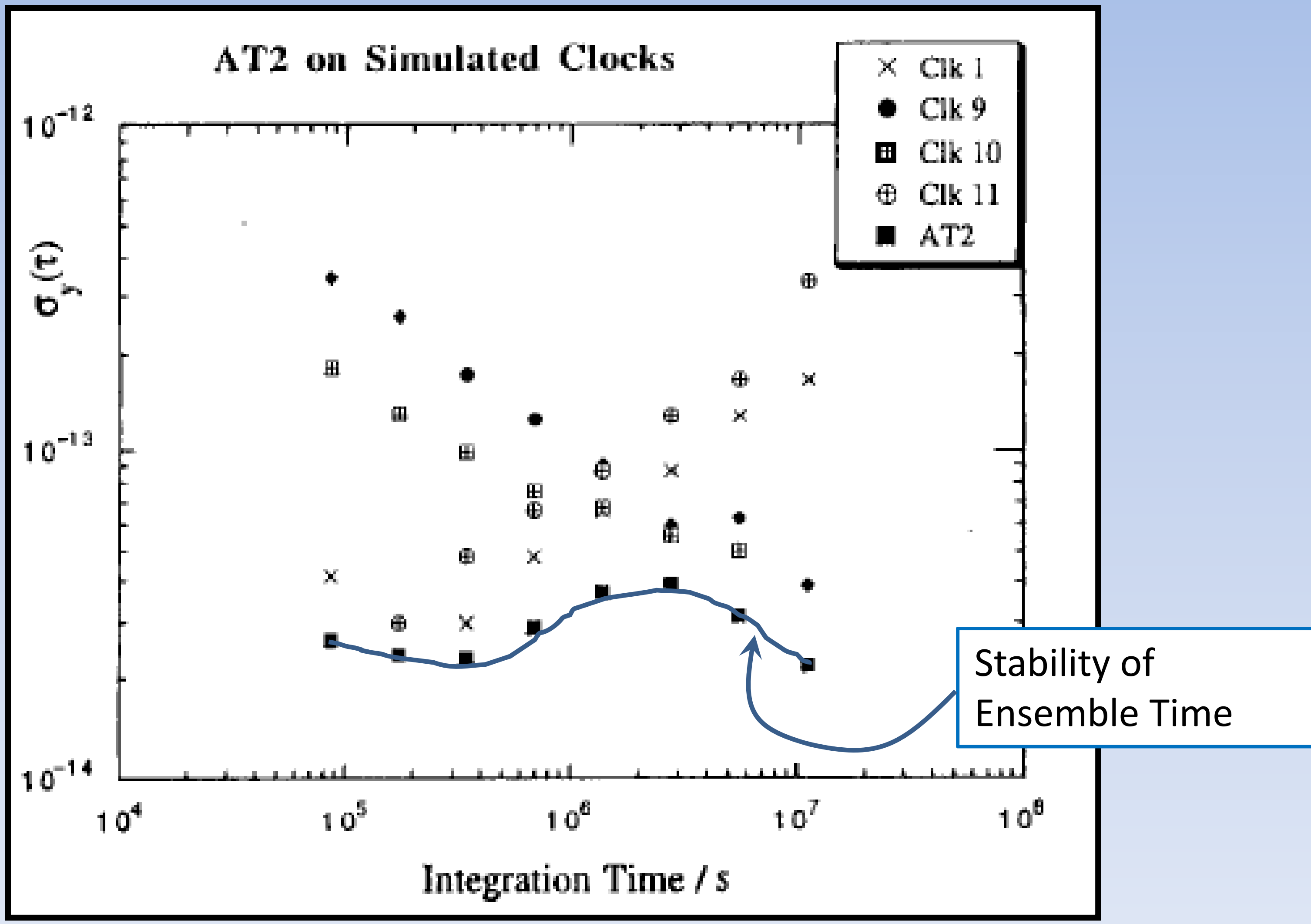
Measures

$\sigma_y(\tau)$	$\sigma_x(\tau)$	$\sigma_y(\tau)$
mod $\sigma_y(\tau)$	$S_x(f)$	mod $\sigma_y(\tau)$
$S_y(f)$	$S_\phi(f)$	$S_y(f)$
$S_\phi(f)$		$S_\phi(f)$

Ensembling two oscillators: phase lock



With Multiple Signals:
Proper Time Scale Algorithm Takes the Best of Each Clock

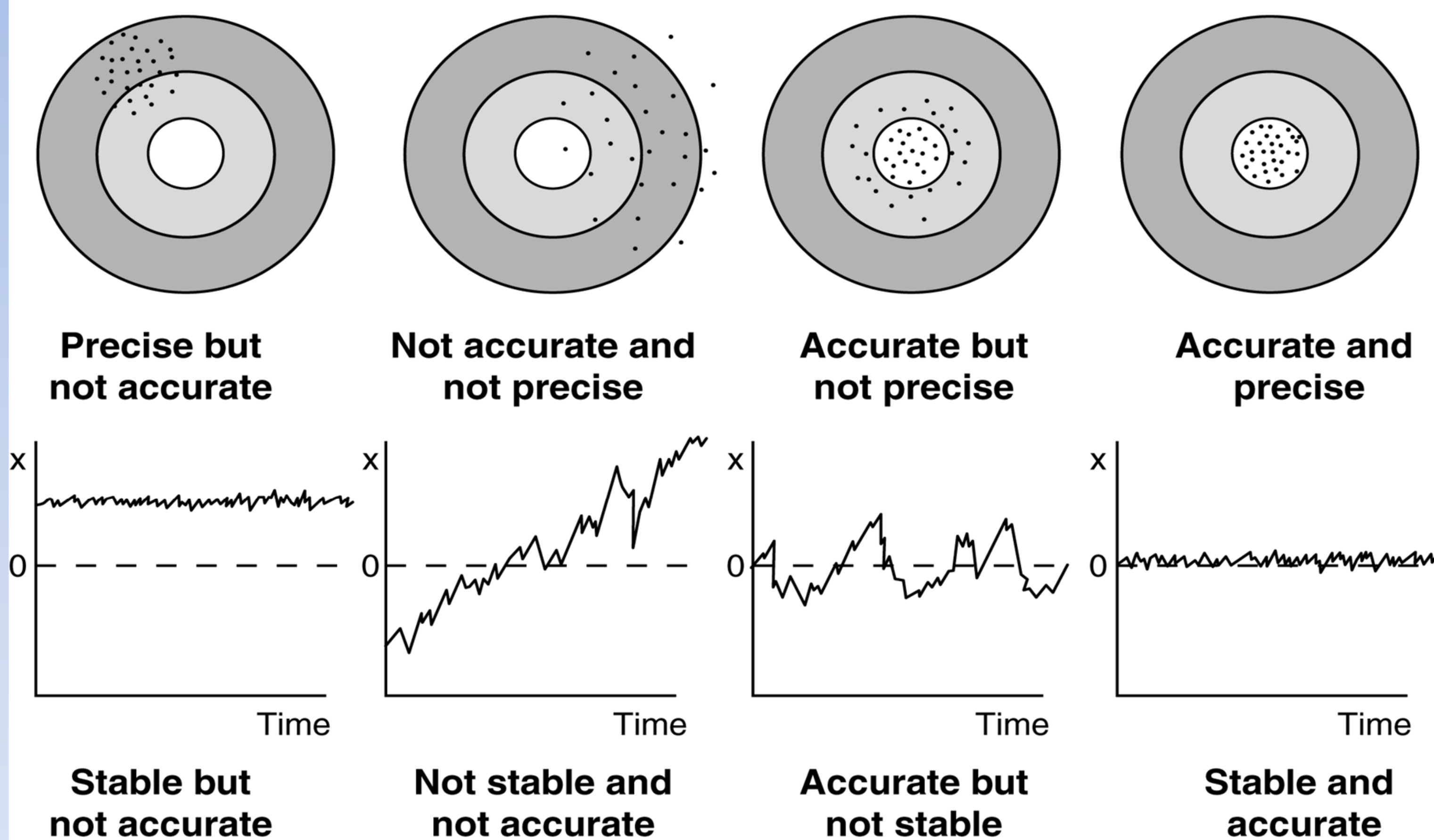


Conclusions

- Determining the dominant noise types allows for optimal prediction
  - Prediction of time error
  - Best estimate of drift
- Optimal Ensemble Time only gets better with more clocks – even a bad clock only improves the ensemble

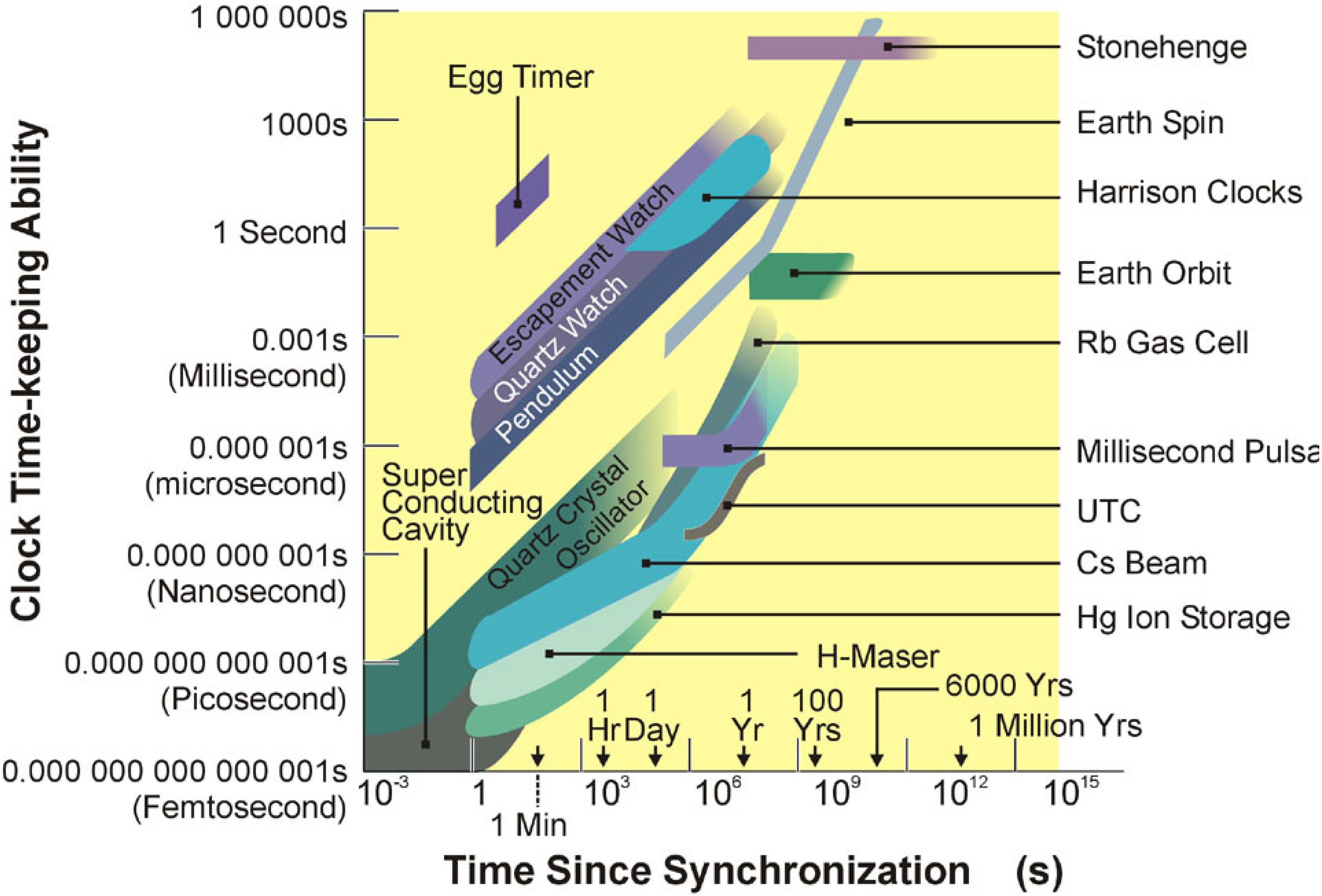
Extra Slides

Accuracy, Precision, and Stability



Adapted from Tutorial on Quartz Crystal Resonators and Oscillators by John R. Vig

Time Dispersion of Various Clocks



MVAR/AVAR

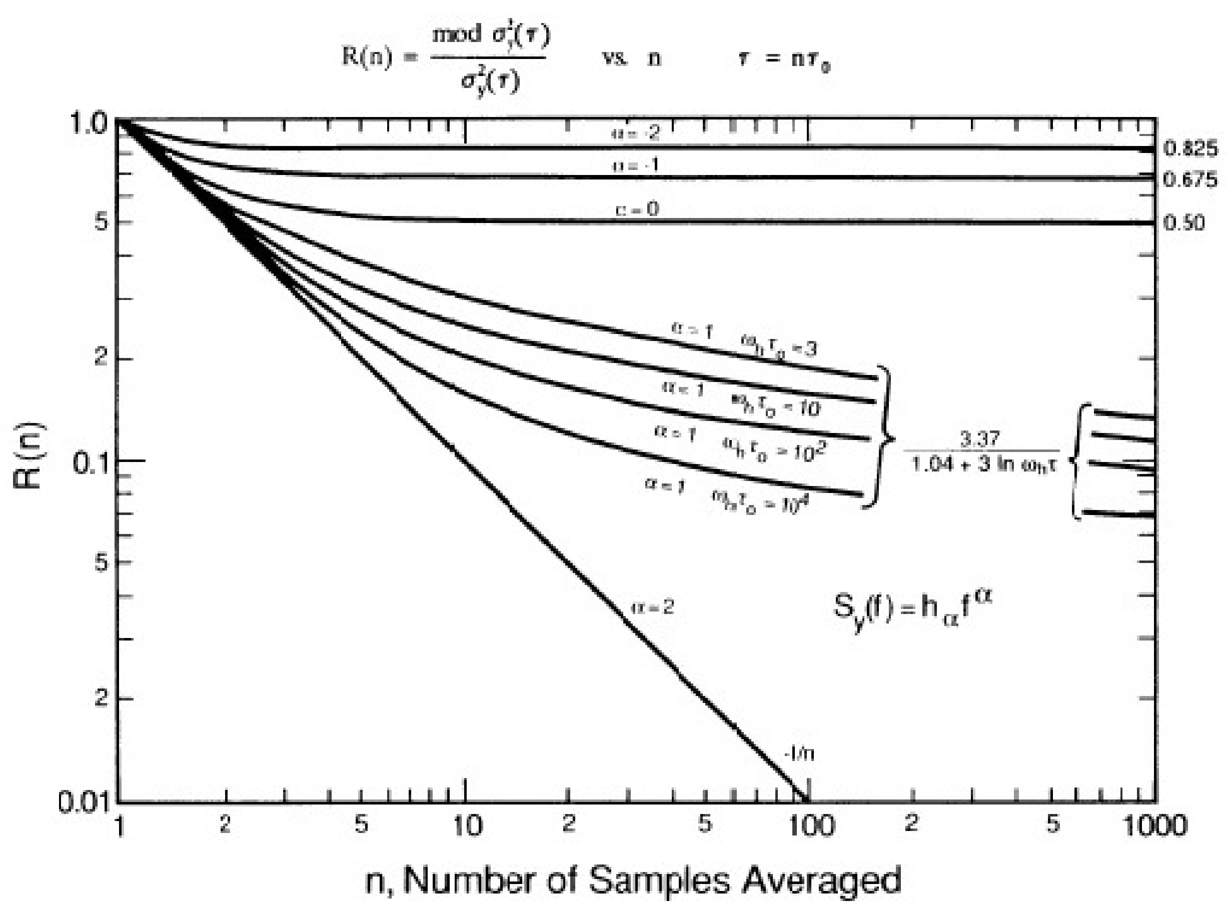


Figure 2. Ratio of mod $\sigma_y^2(\tau)$ to $\sigma_y^2(\tau)$ as a function of the number of time samples averaged together to calculate either variance.