Clock ensembles and time scales

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Steering goal and methods

- Combine data from clocks to generate a local copy of the remote reference time
  - Reference is time from GNSS
  - Reference is UTC from BIPM
  - Reference is UTC(lab) from other source
- Maximize time accuracy of local copy?
- Maximize frequency stability?
- Cost/benefit ratio of process

Why Bother?

- Local copy more reliable
  - Not affected by failure of the link
  - Ensemble can detect clock failure
- Local copy can attenuate link noise
- Statistics of local clocks can detect spoofing attacks
- Remote reference not always available
  - Holdover capability important

Components of a clock

- Oscillator
  - Amplitude= A
  - Output voltage= V
  - Frequency= ω
  - $V = A\sin(\omega t) = A\sin(2\pi f t)$
  - $t_k = \frac{2\pi}{\omega} k = \frac{1}{f} k, \qquad k = 0,1,2,\ldots$
  - $t_{k+1} = t_k + \frac{1}{f}$
- Clock counts ticks from origin by using known oscillator frequency

Characterizing local clocks - 1

- Time, t_k, is specified in seconds, days
  - Day is 86 400 s (24 h X 60 m X 60 s)
    - Modified Julian Day (MJD) is count of days from remote origin so most times are positive
- Time difference, $x_{ij}(t_k)$ is measurement of the difference between devices *i* and *j* in seconds measured at reference time t_k
  - $x_{ij}(t_k) = t_i(t_k) - t_j(t_k)$
- Time interval counter measures x_{ij} mod period
  - Counter starts on t_i and stops on t_j
- If $x_{ij} > 0$ then t_i occurs before t_j

Characterizing local clocks - 2

- Frequency, y_{jr}, is evolution of time difference between clock j and reference clock r. Not the same as ω or *f* in sin *funcn*.
- $y_{jr}(t_k) = \frac{x_{jr}(t_k) - x_{jr}(t_{k-1})}{t_k - t_{k-1}} = \frac{f_j - f_r}{f_r} \cong \frac{\Delta f}{f}$
- Frequency has no units
- Drift is evolution of frequency
- $d_{jr}(t_k) = \frac{y_{jr}(t_k) - y_{jr}(t_{k-1})}{t_k - t_{k-1}}$, units are 1/s
- *y* and *d* usually calculated from *x*

Real-world clocks

- Oscillator has systematic offset from standard, also stochastic variation
  - Physics package
    - Electric, magnetic fields
    - Atom-atom and atom-container collisions
  - Electronics package
    - Voltage offsets and noise in control servo
- Time has stochastic variation from oscillator noise and measurement process
  - Time is ***integral*** of oscillator offset and noise

model of clock

- Frequency, y, changes very slowly in τ
  - Model is piecewise quadratic function of time with almost constant drift term
- $\tau = t_k - t_{k-1}$
- $\hat{x}(t_k) = x(t_{k-1}) + y(t_{k-1})\tau + \frac{1}{2}d(t_{k-1})\tau^2 + \xi$
- $\hat{y}(t_k) = y(t_{k-1}) + d(t_{k-1})\tau + \eta$
- $\hat{d}(t_k) = d(t_{k-1}) + \varsigma$
- $d\tau \ll \eta$; $\tau \ll \frac{\eta}{d}$ and $\varsigma << d_{k-1}$
- ξ, η, ς have mean=0, known variance

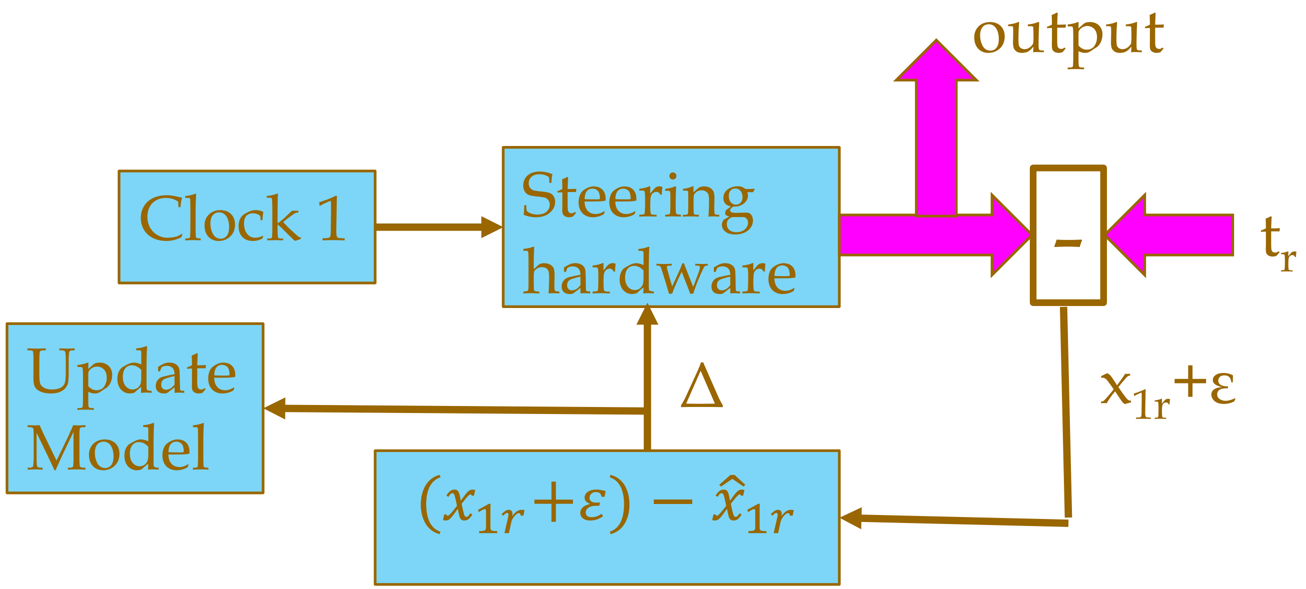
Model of measurements

- $x_{ij}(t_k) = x_{ij}(t_k) + \varepsilon$
- $< \varepsilon >= 0$
- $< \varepsilon^2 >= \sigma^2$, known variance

Statistics of the noise terms

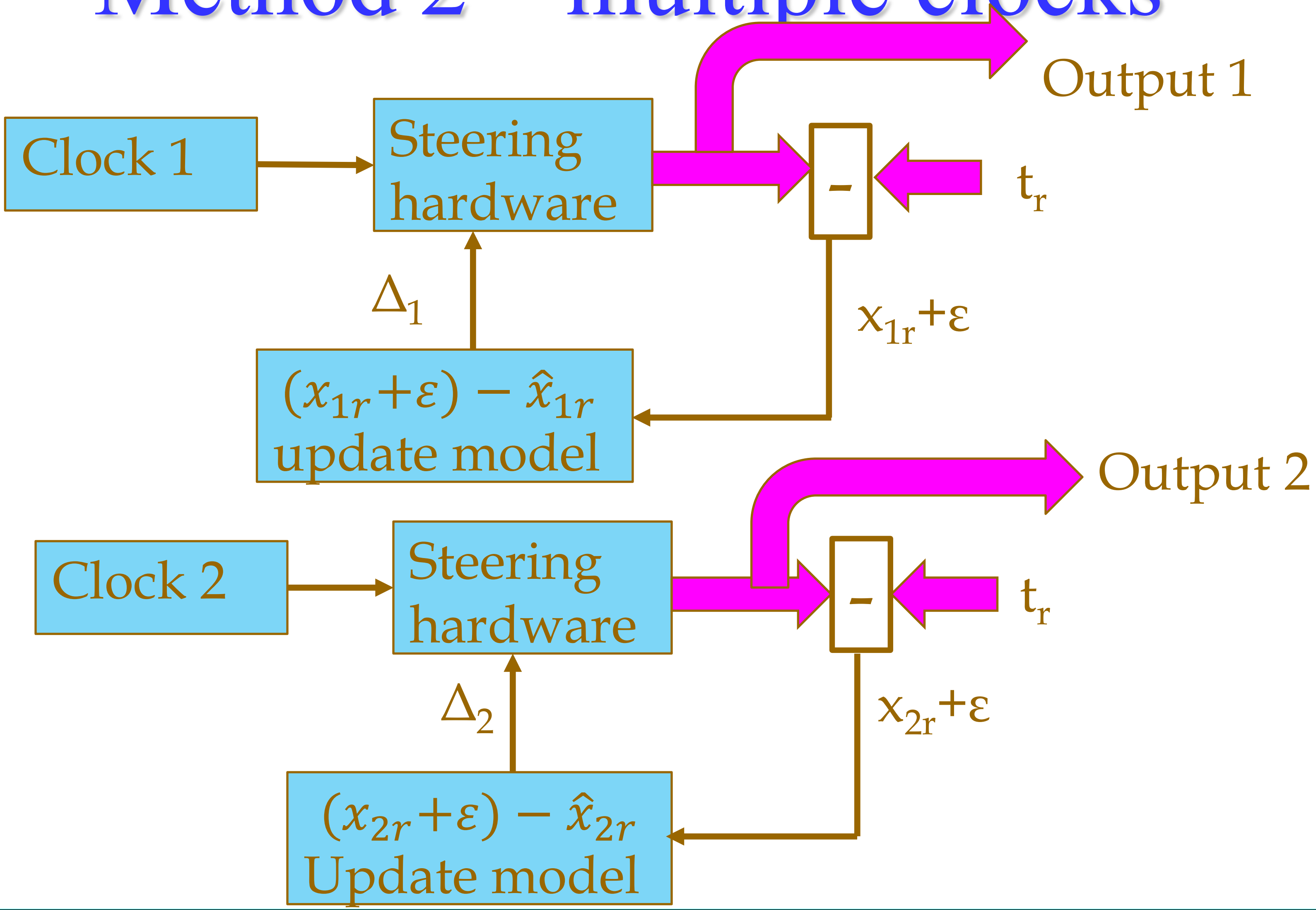
- ε is white phase noise
  - No correlation between consecutive values
- ξ is phase random walk (white freq.)
- η is freq random walk (white drift)
- ς is drift random walk
- Noise terms have mean=0, known variance

Method 1 – single clock



- Provides some hold-over
- Poor error and spoofing detection
- Single point of failure
- Better than nothing

Method 2 – multiple clocks



Method 2 -- continued

- Each clock is independently steered to the reference time
  - Measurement noise affects both channels
- Detects uncorrelated errors, some spoofing attacks
- Better holdover than method 1
- Uncorrelated noise can introduce random walk between channel times

Method 2 -- continued

- Combining outputs
  - Assuming differences are only noise
    - Difficult to verify
    - Combination more stable than either clock
- $\Sigma_1^2 = \frac{1}{\Delta_1^2}; \Sigma_2^2 = \frac{1}{\Delta_2^2}$
- $o = \frac{\Sigma_1^2}{\Sigma_1^2+\Sigma_2^2} o_1 + \frac{\Sigma_2^2}{\Sigma_1^2+\Sigma_2^2} o_2$
- $\Sigma^2(o) = \frac{1}{\sigma^2(o)} = \frac{1}{\Delta_1^2} + \frac{1}{\Delta_2^2}$

The ensemble method - 1

- Choose clock 1 as reference device
- Measure *local* time differences
  - $x_{1j}(t_k) = t_1(t_k) - t_j(t_k)$
- Initialize clock 1 model at time t_k
  - $t_1(t_k) - \hat{x}_{1r}(t_k) = t_r(t_k)$
- Initialize ensemble time using clock 1
  - $t_e(t_0) = t_r(t_k) = t_1(t_k) - \hat{x}_{1r}(t_k)$

The ensemble method -- 2

- Initialize other clock models to match measured clock differences
  - $(t_1(t_k) - \hat{x}_{1r}(t_k)) - \left(t_j(t_k) - \hat{x}_{jr}(t_k)\right) = t_e(t_0) - \left(t_j(t_k) - \hat{x}_{jr}(t_k)\right) = x_{1j}(t_k)$
- Ensemble time via clock *j*
  - $t_e(t_0) = \left(t_j(t_k) - \hat{x}_{jr}(t_k)\right) + x_{1j}(t_k)$

The ensemble method –3

- At the next time step, each clock provides an estimate of the ensemble time by combining its model with measured time difference to clock 1
- $t_e(t_{k+1}) = \left(t_j(t_{k+1}) - \hat{x}_{jr}(t_{k+1})\right) + x_{1j}(t_{k+1})$
  - With $x_{11} = 0$ for all t_k

The ensemble method – 4

- Ensemble time is weighted average of estimates from all clocks
  - No physical clock realizes this average
- Different weighting algorithms:
  - Derived from variances of noise terms
  - Derived from difference between prediction of clock and ensemble mean at each epoch
- Prediction error of each clock used to update its model
  - Models define paper ensemble as offset from physical clocks

Comments - 1

- Calculation of weights is circular
  - Prediction error is systematically too small
    - Weights are systematically too large
      - Weight of good clock can approach 1
- Maximum weight limited
  - AT1: max weight = 30%
- Limiting weight of good clock gives too much weight to other clocks

Update clock models

- Update models from prediction error
  - Models have 6 parameters
    - No unique method
- Algorithm based on time *differences*
  - *Overall time and frequency are free parameters*
- *Ensemble attenuates frequency noise of clocks*
  - *Ensemble can improve frequency stability*
- *Ensemble has overall random walk in time (or worse) from residual frequency noise*
  - *Must be limited by external calibration data*

Summary

- Ensemble "remembers" initial calibration subject to clock noise
- Ensemble detects outliers and has no single point of failure
- Ensemble attenuates noise of member clocks
- Clock model integrates residual noise
  - Random walk in time relative to reference

References

- The statistical modeling of atomic clocks and the design of time scales
  - Judah Levine, Rev. Sci. instr. *83*, 021101 (2012)
- A review of reduced Kalman filters for clock ensembles
  - Charles A. Greenhall, IEEE Trans. UFFC *59*, 491 (2012)
- On the use of Kalman filters in time scales
  - L. Galleani and P. Tavella, Metrologia, *40*, S326, (2003)